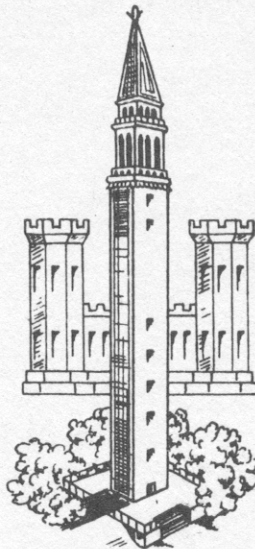


SECOND APPROXIMATION TO THE SOLUTION OF THE SUSPENDED LOAD THEORY

By

H. A. Einstein and Ning Chien



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CORPS OF ENGINEERS SEDIMENT STUDIES PROGRAM

FOR MISSOURI RIVER BASIN

MISSOURI RIVER DIVISION

FORT PECK DISTRICT

OMAHA DISTRICT

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KANSAS CITY DISTRICT

The Corps of Engineers Missouri River Basin sediment studies program was established for the development of practical sediment engineering for rational evaluation, regulation, and utilization of fluvial sediment phenomena. It was implemented as a comprehensive, basin-wide program for coordination of studies of sediment problems in the overall basin program for flood control and allied purposes as well as for continuity and perspective in the planning and design of individual projects. The program includes both investigations for the development of sediment transport theory and observations of existent and occurring phenomena for the purpose of developing the applications of theory to practical problems, developing empirical relationships, and providing aids to judgment.

The program has been conducted during the tenures of and supported by the following Division Engineers:

Lieutenant General Lewis A. Pick

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Mr. F. B. Slichter was Chief of the Engineering Division from the inception of the program until April 1949. Mr. W. E. Johnson has been Chief of the Engineering Division since that time. The program was formulated and organized by Mr. R. J. Pafford, Jr., Chief, Planning and Reports Branch. Planning and execution is under the immediate direction of D. C. Bondurant with technical advice and assistance provided by an Advisory Board consisting of:

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Introduction

Based on the mechanics of turbulent flow of fluids, it has been established that the distribution of the relative concentration of the suspended load is of the following form:

$$\frac{C_y}{C_a} = \left(\frac{d-y}{y} \frac{a}{d-a} \right)^Z \quad (1)$$

with

$$Z = \frac{V_s}{K u_*} \quad (2)$$

where C_y and C_a are the concentrations of a given grain size with settling velocity V_s at the distance y and a , respectively, from the bed, K is Karman's universal constant (0.4 for clear water in pipes), u_* is the shear velocity at the bed, and d is the depth of flow.

Eq. (1) has been shown to be the correct form of the distribution function in large rivers, except that the value of the exponent Z given by eq. (2) does not agree with Z_1 , the exponent that fits the measured data. In general, the distribution of the suspended load is more uniform than that indicated by the theory. Vanoni (1946) and Ismail (1951) proposed that the sediment transfer coefficient, ϵ_s , is not the same as the momentum transfer coefficient, ϵ_m , but that they are related by the following formula:

$$\epsilon_s = \beta \epsilon_m \quad (3)$$

where β seems to be a function of the particle size. And

$$Z_1 = \frac{V_s}{\beta K u_*} = \frac{Z}{\beta} \quad (4)$$

which indicates the deviation of Z_1 from Z . On the other hand, Anderson (1942) has shown from the results of river measurements that

(1) for small Z -values, the measured exponent Z_1 is the same as that indicated by equation (2); and

(2) for large Z -values, the measured values of Z_1 are smaller than Z and seem to approach a finite value.

Recently the Corps of Engineers have conducted two sets of more accurate measurements in the Missouri River at Omaha, Nebraska, and in the Atchafalaya River at Simmesport, Louisiana. Figure 1 gives the results. The shear velocity at the bed, u_* , is determined from the depth of flow and water surface slope. The constant k is then determined from the semi-logarithmic velocity profile by the following equation:

$$k = 2.3 u_* S \quad (5)$$

where S is the slope of the semi-logarithmic velocity profile $d(\log y)/du$. The settling velocity of sediment, V_s , is taken from Rubey (1933) for the analysis of the Atchafalaya River data, and from the settling velocity-grain size curve provided by the Corps of Engineers, Missouri River Division, in analyzing the Missouri River data. Both settling velocity data are determined in still water. From the values of u_* , k , and V_s , the exponent Z is calculated according to equation (2). The corresponding Z_1 is determined

from the measured distribution of particle concentration for that size. Each pair of Z_1 and Z is then plotted in Figure 1. The Missouri River data are obtained from both the central flat area and the rough area near the right bank of a straight reach about one mile long, and 1,000 feet wide. Only the results taken from the flat area are used in this analysis. The topographic map of the flat area reveals no appreciable sand bars or other irregularities on the river bottom in the neighborhood of the measuring stations. The Atchafalaya River data are obtained from the Simmesport-Melville Reach where the average depth varies from 36 to 52 feet for discharges of 120,000 to 415,000 cfs. There are ten sets of sediment distribution measurements and eleven sets of velocity distribution measurements available. Unfortunately the two measurements are not taken at the same time and from the same vertical. Only measurements which are conducted not more than two or three days apart and during times of constant river discharge, are paired together and plotted in Figure 1. The results thus obtained seem to correlate well with the Missouri River data. Figure 1 materially substantiates the findings of Anderson. The scatter of the measured points is large, yet the trend is evident. Not only Z_1 is smaller than Z , but the deviation between these two also becomes consistently larger as Z increases. It is rather doubtful that this discrepancy can be attributed to the difference between C_s and C_m alone. The supposition that the value β , which is the ratio of sediment transfer coefficient to momentum transfer coefficient, remains constant in the vertical also needs some justifications.

The failure of the suspended load theory to describe river conditions has long been a challenge to the workers on sedimentation. In cooperation

with the U. S. Corps of Engineers, Missouri River Division, Omaha, Nebraska, efforts have been made recently by the writers to attack this perplexing problem, and some of the results are presented in this paper. A review of the existing suspended-load theory is made at first and the assumptions involved in the derivation of the theory are pointed out. Possible improvements on these assumptions which lead to results comparable to those given by the river measurements are then presented in detail. The writers do not attempt to claim that the methods presented in this paper will be the final solution. In fact, these methods must be classed as new approximations to the solution of the suspended-load theory. However, the writers do hope that this paper will stimulate enough attention and point research in the direction in which the final solution is possible.